Implementation of the intensity and frequency dependent shallow foundation element in OpenSees

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ABSTRACT

Comprehensive understanding of Soil-Structure Interaction (SSI) of shallow foundation and its effect on the overall performance of the structure subjected to a seismic load is important in accurate seismic performance assessment. The SSI is often modelled with computationally expensive numerical models considering the material nonlinearity of the soil and geometric nonlinearity of the foundation uplift. Different approaches have been proposed in the past to simplify the analysis with lumped spring models, and constitutive models that can replicate nonlinear behavior of soil-foundation system. In order to consider the frequency-dependent behavior of soil domain, various transformation methods have also been used to convert frequency-dependent behavior of the soil domain to time domain analysis. These simplified models, however, could not capture both the intensity and frequency dependency of soil-foundation system.

Recently, the authors developed a simplified method which can capture both the nonlinearity and frequency dependency of soil-foundation system in time domain analysis. The paper aims to expand the application of this method by implementing the proposed model into the widely used software, OpenSees. The methodology in incorporating frequency-dependent behavior into a nonlinear time domain analysis using convolution coefficients are discussed. In addition, the implementation in OpenSees is verified against the implementation in MATLAB. As a verification of the implementation, a bridge pier is analyzed with shallow foundation subjected to a seismic excitation where a frequency-dependent linear elastic soil, nonlinear soil, and nonlinear soil with uplift of the foundation cases are covered. The development presented in the paper provides accessible element for users to incorporate intensity and frequency dependent behavior of soil foundation system in the overall seismic performance of structures with significant reduction in the model complexity and analysis time of the soil domain.

Keywords: Soil structure interaction (SSI), Intensity and frequency dependent, shallow foundation, seismic analysis of soil foundation, OpenSees.

INTRODUCTION

An accurate assessment of soil structure interaction (SSI) system in seismic analysis is pivotal in evaluating overall performance of structure subjected to earthquakes. The nonlinear and frequency-dependent behavior of shallow foundation in seismic excitation is paramount in capturing an accurate behavior of the soil and foundation interaction. The conservative approach in seismic design of foundation system generally neglect inelastic response of foundation system. In most of the conservative design cases, the foundation is designed to undergo limited nonlinear response with limited permanent deformation. This approach, however, may lead to oversizing of the foundation and prevent foundation to take advantage of its energy dissipation capacity [1]. Many researchers have been investigating on the ability of the shallow foundation to dissipate energy through inelastic response during earthquakes which reduce the seismic demand on the super-structure. There are many simplified models for capturing nonlinear behavior of soil-foundation system such as Beams on Winkler type foundation [2], lumped spring models [3], and constitutive models. One of the constitutive model, which is referred to as macroelement, have been developed by Chatzigogos et al [4][5] to capture the response of shallow foundation with uplift and nonlinear behavior of soil with lumped node. Macroelement provides an efficient model with relatively small number of required parameters to analyze the nonlinear behavior of soil foundation system. The analysis results using macroelement show the beneficial effect of nonlinear response of foundation to the structure, which provides substantial contribution to overall energy dissipation during seismic excitation when compared to fixed-base foundation. ASCE 41-13 has introduced allowable uplift of shallow foundation in the seismic design of shallow foundation through extensive experimental result as well [6][7].

The importance of frequency-dependency of SSI effects has been studied by Wolf et al [8] [9] with the use of transformation method to convert frequency dependent characteristics in time domain analysis. One of the main challenges in incorporating frequency-domain analysis to time domain analysis have been the stability of the transformation method [10]. Also, the underlying assumption behind the transformation have been linear superposition of convolution terms which cannot be used in nonlinear analysis of the system. The authors have proposed a modelling method for shallow foundation in which the nonlinear behavior and frequency-dependent behavior are integrated [11]. The incorporated method was improved by Lesgidis et al. [12] where multi-objective algorithm is used to provide accurate response of nonlinear frequency dependent analysis.

As the research is progressing towards accurate assessment of SSI effect subjected to seismic excitation, the authors have implemented the algorithm to the open source code, OpenSees, which is widely used in earthquake engineering. The element allows users to analyze soil-foundation system with nonlinear frequency-dependent behavior of soil with the use of macroelement [4] and Nakamura's coefficient terms [13]. This paper provides details of the element implementation: the input parameters, outline of the element, program flow, and results are presented. The verification example compares the analysis results from OpenSees and FEM model which were presented in Chai et al. [11]. The models provide wide range of intensity and frequency dependency with sinusoidal excitation to verify the nonlinearity and frequency dependency of the element as intended. The element is also verified with earthquake excitation on a bridge pier model.

INTEGRATION MODEL

The algorithm of integration of intensity and frequency dependent elements was first introduced by Chai et al [11]. In the integration method, the equation of motion of the soil-foundation system is defined as Eq.(1):

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{c}\dot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = \boldsymbol{F} \tag{1}$$

where $\boldsymbol{m}$, $\boldsymbol{c}$, and $\boldsymbol{k}$ are mass, stiffness, and damping matrices of the soil-foundation system, and $\boldsymbol{F}$ is the excitation force. The system of equation can then be expressed in frequency domain as shown in Eq.(2).

$$\begin{gathered}(\boldsymbol{k} - \omega^2\boldsymbol{m} + i\omega\boldsymbol{c})\boldsymbol{u}(\omega) = \boldsymbol{F}(\omega) \\ \text{or, } \boldsymbol{K}(\omega)\boldsymbol{u}(\omega) = \boldsymbol{F}(\omega)\end{gathered} \tag{2}$$

where $\boldsymbol{K}(\omega) = \boldsymbol{k} - \omega^2\boldsymbol{m} + i\omega\boldsymbol{c}$. Eq.(2) can be partitioned with two separate domains; foundation and soil represented with subscripts, *f* and *s*, respectively as shown in Eq.(3).

$$\begin{bmatrix}\boldsymbol{K}_{ff} & \boldsymbol{K}_{fs} \\ \boldsymbol{K}_{sf} & \boldsymbol{K}_{ss}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}_f \\ \boldsymbol{u}_s\end{bmatrix} = \begin{bmatrix}\boldsymbol{F}_f \\ \boldsymbol{F}_s\end{bmatrix} \tag{3}$$

The soil-foundation system subjected to dynamic load is illustrated in Figure 1.

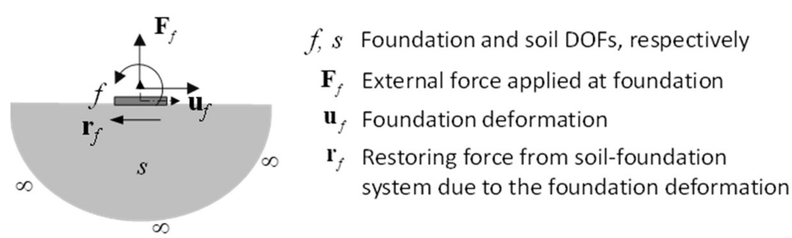


Figure 1. Schematic diagram of soil-foundation system with dynamic load at foundation

If the seismic excitation is applied as equivalent dynamic force to a structural system (i.e. no dynamic force is applied to soil domain, $\boldsymbol{F}_s = 0$). Then, the Eq.(3) can be condensed to the following expression.

$$\begin{gathered}\left(\boldsymbol{K}_{ff} - \boldsymbol{K}_{fs}\boldsymbol{K}_{ss}^{-1}\boldsymbol{K}_{sf}\right)\boldsymbol{u}_f = \boldsymbol{F}_f \\ \text{or, } \boldsymbol{K}_f\boldsymbol{u}_f = \boldsymbol{F}_f\end{gathered} \tag{4}$$

where $\boldsymbol{K}_f$ is a dynamic stiffness of soil-foundation system which is a function of excitation frequency, ω. The restoring force in Eq. (4), $\boldsymbol{F}_f$, is dependent on dynamically changing displacement $\boldsymbol{u}_f$ where soil-foundation system is assumed to be linear elastic. The symbol, $\boldsymbol{r}_D$, will be used to represent the restoring force, $\boldsymbol{F}_f$, which results from dynamic interaction.

If soil-foundation system behaves in the inelastic range due to local bearing failure or uplift, Eq. (4) cannot be used to evaluate the restoring force because it is based on linear-elastic assumption. The restoring force resulting from inelastic behavior, but not dynamic response, is represented with a symbol $\boldsymbol{r}_S$. Thus, if the system is linear elastic and the excitation is applied in static manner (i.e. $\omega = 0$), then $\boldsymbol{r}_D$ and $\boldsymbol{r}_S$ are equal to $\boldsymbol{k}_f\boldsymbol{u}_f$ where $\boldsymbol{k}_f$ is a condensed static stiffness matrix of the soil-foundation system.

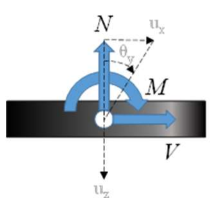
Thus, Chai et al. [11] proposed that the overall soil-foundation system which includes both dynamic elastic and quasi-static inelastic responses can be expressed with Eq.(5).

$$\boldsymbol{r}_f(\boldsymbol{u}_f) = \boldsymbol{r}_D(\boldsymbol{u}_f) + \boldsymbol{r}_S(\boldsymbol{u}_f) - \boldsymbol{k}_f\boldsymbol{u}_f \tag{5}$$

Note that at low excitation frequency, $\boldsymbol{r}_D(\boldsymbol{u}_f)$ and $\boldsymbol{k}_f\boldsymbol{u}_f$ cancel each other, resulting in quasi-static inelastic behavior of soil. In the same way, when amplitude of displacement is small (i.e. linear elastic behavior), $\boldsymbol{r}_S(\boldsymbol{u}_f)$ and $\boldsymbol{k}_f\boldsymbol{u}_f$ cancel each other. The authors have used macroelement proposed by Chatzigogos et al. [4] to represent the nonlinear behavior of soil, and a convolution integration by Nakamura [13] to represent the dynamic stiffness in time domain. Chai et al. [11] states that integration method can also be used with other constitutive models and transformation method to represent dynamic stiffness. The following sections briefly introduce the macroelement [4] and convolution integral [13] which represents $\boldsymbol{r}_S(\boldsymbol{u}_f)$ and $\boldsymbol{r}_D(\boldsymbol{u}_f)$, respectively.

Macroelement

The following is a short summary of the macroelement developed by Chatzigogos et al [5]. Macroelement is a single lumped node model with three DOFs (horizontal, vertical, and rotation) to describe the quasi-static nonlinear behavior of soil foundation system. Figure 2 shows the foundation DOFs of the macroelement model and its generalized force-displacement relationship.



$$\boldsymbol{Q} = \begin{bmatrix} Q_N \\ Q_V \\ Q_M \end{bmatrix} = \begin{bmatrix} \dfrac{N}{N_{max}} \\ \dfrac{V}{N_{max}} \\ \dfrac{M}{DN_{max}} \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} q_N \\ q_V \\ q_M \end{bmatrix} = \begin{bmatrix} \dfrac{u_z}{D} \\ \dfrac{u_x}{D} \\ \theta_y \end{bmatrix} \longrightarrow \begin{bmatrix} \dot{Q}_N \\ \dot{Q}_V \\ \dot{Q}_M \end{bmatrix} = \begin{bmatrix} K_{NN} & 0 & K_{NM} \\ 0 & K_{VV} & 0 \\ K_{MN} & 0 & K_{MM} \end{bmatrix} \begin{bmatrix} \dot{q}_N{}^{el} \\ \dot{q}_V{}^{el} \\ \dot{q}_M{}^{el} \end{bmatrix}$$

Figure 2. Generalized force and displacement relationship diagram

where $Q_N, Q_V, and\ Q_M$ are forces in vertical, horizontal, and rotational DOF respectively. The terms $q_N, q_V, and\ q_M$ represent displacements in vertical, horizontal, and rotational DOF respectively. The force parameters are normalized with maximum bearing capacity of foundation, N_{max}, and displacements are normalized with characteristic dimension of the footing, *D* (i.e. width of the strip foundation or radius of circular foundation) for analysis purposes.

Based on a few assumptions regarding uncoupled behavior of shallow foundation on horizontal and rotational DOFs, the generalized force displacement relationship can be simplified to the expression in right side of Figure 2 with uplift condition. Note that displacement vectors for uplift are expressed with terms with the superscript, ^{el}, to indicate geometric uplift condition which occurs at the linear elastic analysis. In this formulation, the generalized force and displacements are expressed in the incremental format, denoted by dots on top of each variables. Moment of uplift initiation is given by Eq. (6).

$$Q_{M,0} = \pm \frac{1}{\alpha} Q_N \tag{6}$$

Based on this uplift condition, uplift stiffness can be evaluated as shown in Eq.(7) for the coupled vertical and rotational DOFs. The suggested values for uplift parameters for strip and circular foundation are provided as shown in Table 1.

$$K_{MN} = K_{NM} = \begin{Bmatrix} 0, & if\ |q_{m,o}{}^{el}| \le |q_{M,o}{}^{el}| \\ \frac{1}{2}K_{NN}\left(1 - \frac{q_{M,o}{}^{el}}{q_M{}^{el}}\right), & if\ |q_{M,o}{}^{el}| > |q_{M,o}{}^{el}| \end{Bmatrix} \quad KMM = \begin{Bmatrix} KMM, & if\ |q_{m,o}{}^{el}| \le |q_{m,o}{}^{el}| \\ KMM\left(\frac{q_{m,o}{}^{el}}{q_m{}^{el}}\right)^2 + \frac{1}{4}KNN\left(1 - \frac{q_{m,o}{}^{el}}{q_m{}^{el}}\right)^2, & if\ |q_{m,o}{}^{el}| > |q_{m,o}{}^{el}| \end{Bmatrix} \tag{7}$$

The mechanism of soil material yielding in the vicinity of the footing is described by the ellipsoidal bounding surface through a hypoplastic model. The development of plastic displacement is calculated based on mapping rule where each point inside the bounding surface is projected onto the bounding surface which explains the ratio of the current force experienced by the foundation to the maximum bearing capacity of the foundation. The expression which describes this plasticity in macroelement is shown in Eq.(8).

Table 1. Numerical parameters for uplift model for strip and circular foundation

Types	**α**	**β**	**γ**
Strip	4	2	½
Circular	6	3	¾

$$h = \left(p1 * K_f\right) ln\left(\frac{\lambda^{p2+1}}{\lambda_{min}{}^{p2}}\right) \tag{8}$$

where h represents plasticity stiffness of the macroelement. The $p1$ parameter in Eq.(8) is a factor used to describe nonlinear behavior of soil upon loading and is often assumed to be 0.1. K_f is the static stiffness of the foundation and λ is the ratio of current force step to the bounding surface. The p2 parameter is used to describe behavior of unloading, has suggested a value in the range from 3 to 5 for cohesive soil. More information on equations used for macroelement is provided in Chatzigogos et al [4].

This function is not restrictive to lognormal relation, as there may be other functions that could potentially improve or replace the lognormal relation per different characteristics or different types of soil, but for cohesive soil the function provides simple yet sufficiently accurate results than other constitutive models for nonlinear behavior of soil [5].

Nakamura's coefficient terms

In Nakamura's model, the restoring force from dynamic response of soil-foundation system is defined using state parameters such as stiffness, damping, and mass terms [13]. The expression of Nakamura's transformation method can be expressed using these state parameters as coefficients as shown in Eq.(9).

$$\boldsymbol{r}_D(t) = \boldsymbol{K}_0\boldsymbol{u}(t) + \boldsymbol{C}_0\dot{\boldsymbol{u}}(t) + \boldsymbol{M}_0\ddot{\boldsymbol{u}}(t) + \left\{\sum_{j=1}^{N-1}\boldsymbol{K}_j\boldsymbol{u}(t-t_j) + \sum_{j=1}^{N-2}\boldsymbol{C}_j\dot{\boldsymbol{u}}(t-t_j)\right\} \tag{9}$$

where $\boldsymbol{r}_D(t)$ is the restoring force occurring from the soil at time t. $\boldsymbol{K}_0$, $\boldsymbol{C}_0$ and $\boldsymbol{M}_0$ represent the instantaneous stiffness, damping, and mass of soil foundation system at time t, respectively. $\boldsymbol{K}_j$ and $\boldsymbol{C}_j$ represent the recursive parameters of the past displacement and velocity terms. Using this transformation method with state parameters defined above allows Eq.(9) can be integrated into a typical numerical time integration scheme, such as Newmark's method. More information is available in details for Nakamura's convolution integrals [13] and implementation of the method into integration model by Chai et al. [11].

IMPLEMENTATION OF THE ELEMENT

From the integration model, the required parameters for each element (macroelement and Nakamura's model) is discussed. In OpenSees implementation, the element macroel_freq requires 8 parameters in order to analyze inelastic frequency-dependent behavior of soil-foundation system. The list of parameters along with description of the parameters are shown below.

```
element macroel_freq $ele Tag $iNode $jNode $KNN $KVV $KMM $B $Alpha $p1
$p2 <–file 'file_name.txt'>
```

$eleTag	unique element object tag
$iNode $jNode	end nodes
$KNN*	Vertical Stiffness of footing
$KVV*	Horizontal Stiffness of footing
$KMM*	Rotational Stiffness of footing
$B	Foundation width (m)
$Alpha	Uplift condition for foundation (4 for strip foundation, 3 for circular foundation)
$P1	plastic parameter for virgin loading
$P2	plastic parameter for unloading/reloading
-file	.txt containing dynamic impedance of soil-foundation system (can be obtained from theoretical equation or from other software

Note: * values can be replaced with theoretical solutions, thus required parameters could be Reduced to shear modulus, density, and Poisson's ratio in soil with theoretical solution available.

The overall flow chart of the macroel_freq analysis is illustrated in Figure 3.

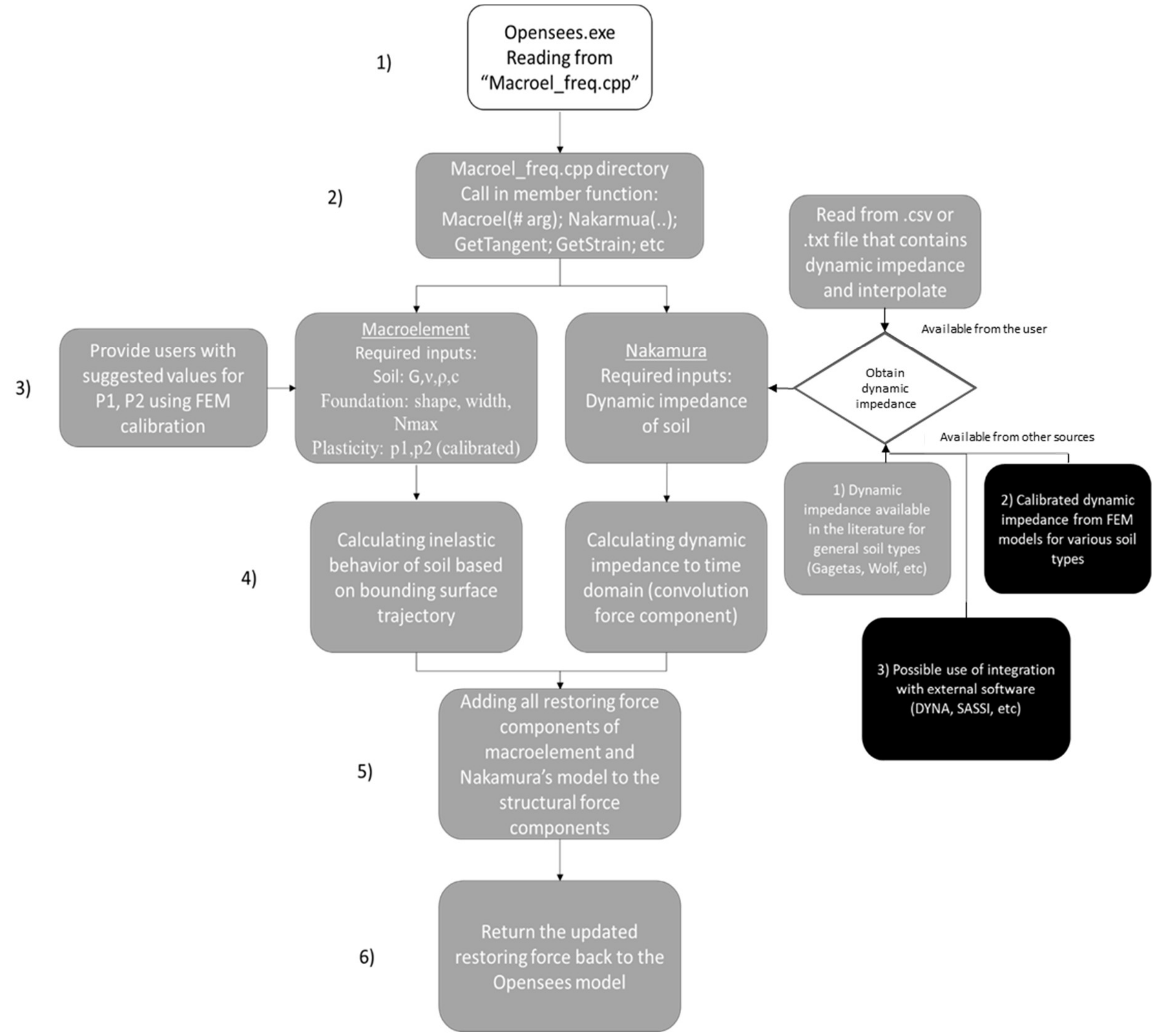


Figure 3. Flow chart of macroel_freq element in OpenSees

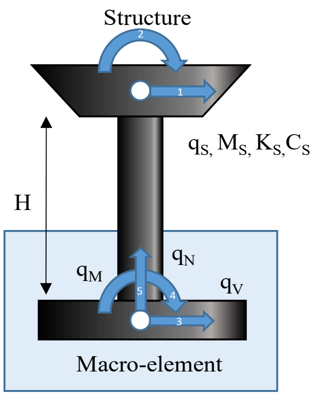
As shown in Figure 3, the analysis is carried out where the left side of the chart shows calculation of macroelement while the right side of the flow chart shows convolution coefficients from Nakamura's model. For macroelement analysis, the required parameters are listed and suggested values for specific soil types will be provided for the users. Once the parameters are defined, the macroelement calculates the quasi-static inelastic behavior of soil based on the bounding surface trajectory. On the right side of the flow chart, Nakamura's coefficient terms are calculated based on user-defined dynamic impedance. As shown in the flowchart, dynamic impedance can be manually defined by users through external text file. Theoretical dynamic impedance functions can be used from literature references available from Chatzigogos et al. [4] to obtain dynamic impedance in text format in case users do not have pre-defined dynamic impedance of soil. The black blocks illustrate potential development of the element to incorporate communication with external software for users to define dynamic impedance of soil from other software such as LS-DYNA [15], SASSI [16] or other FEM models. Once all the required parameters are obtained from a user, the element then calculates the restoring force with macroelement and Nakamura's coefficients simultaneously.

The challenge in incorporating this integration within OpenSees lies in nonlinear solution scheme and transformation of convolution coefficients used for frequency dependency of soil. Macroelement requires iteration at each time increment to obtain converged solution of inelastic response of soil. On the other hand, Nakamura's convoluted force terms are calculated based on converged state parameters of the time step. In the context of implementation procedure in OpenSees, macroel_freq::setTrialStrain is a class member function that allows element to update the imposed displacement for applied force. If the element yields nonlinear behavior, setTrialStrain function is repeated until converged displacement for the applied force is obtained. At each iteration, Nakamura's restoring force, which is calculated based on Nakamura's coefficients multiplied by previously committed state parameters (displacement and velocity), is added onto the restoring force equation for next iteration. Since iteration only changes macroelement portion of the analysis which is nonlinear, Nakamura's restoring force remains the same throughout iteration until analysis step moves onto next time increment.

All the local variables are deleted after the setTrialStrain function ends. Thus, it is important to utilize dynamic memory allocation to store any parameters within the function for external use outside of the function. In this case, Nakamura's restoring force would be a great example of utilizing dynamic memory allocation. At each iteration, previous state parameters (displacement and velocity terms) are required for calculating Nakamura's restoring force. In OpenSees, after converged displacement is obtained, the element records the results, updates the analysis to next analysis step and the refreshes for next analysis step. However, in this integration model, as previous state parameters are required in the analysis, all of the previous analysis results are stored for accumulated time step using memory allocation and restoring force is calculated from Nakamura's coefficient terms. Once overall analysis is completed, the parameters are no longer required and the memory should be deleted. Otherwise, memory leakage occurs and cause OpenSees to crash.

VERIFICATION MODEL

The verification model used in this paper is from Chai et al [11]. The verification model consists of a bridge pier with 10 m wide rigid strip foundation resting on 100 m wide by 100 m depth soil domain. A dynamic impedance of soil domain is obtained using FE model that has with 1 m four-node quadrilateral elements. In order to verify that FE model with Lysmer and Kuhlemeyer [14] energy absorbing boundary produces correct impedance functions, two analysis programs, OpenSees and RS2 [17], are used for the modelling and verification. The dynamic impedance function from both programs are in good agreement. For the uplift cases, OpenSees is used for analyzing foundation with gap interface element. More details of the model properties and geometry is shown in Figure 4 and also can be found in Chai et al [11].



Numerical parameters	Properties
Structure	$M_s = 5 \times 10^4$ kg; $J_s = 1.25 \times 10^6$ kg m^2; H = 20m; S = 1.6 m^2, I = 2.13 m^4; E = 35 x 10^9 Pa
Foundation	B = 10m; e = 0.7 m; L = 1.0m; $\rho = 18 \times 10^3$ kg/m^3; E $\rightarrow \infty$
Interface	Uplift allowed; no sliding
Soil	ρ = 1.6 ton/m^3, G = 65,000 kPa v = 0.25, c = 30 kPa

*Note: For strip foundation, unit length of 1 meter is applied.

Figure 4. Model description with geometry and material properties

The above problem can be modeled using the macorel_freq element with the following parameters shown in Table 2.

Table 2. Parameters for the vefrification model with macroel_freq element

element	eleID	node1	node2	KVV	KNN	KMM	B	alpha	p1	p2
macroel_freq	1	1	2	201532	92141.3	3224290	10	4	5000	1

Sinusoidal rotation is applied at the foundation and the restoring moment at the foundation is obtained. The obtained hysteretic behavior is verified with results from the implementation in MATLAB. The load conditions of varying intensity of harmonic excitation ranging from 500 kN·m to 2000 kN·m with range of frequencies of excitation from 5 Hz to 20 Hz applied at the foundation are analyzed. Figure 5 shows verification results obtained with nonlinear soil and frequency dependency of soil and Figure 6 shows verification results with addition of uplift of the foundation taken into consideration. For nonlinear analysis without uplift, the OpenSees element and MATLAB implementation shows good agreement. For the case with uplift, there is slight difference at low frequency with varying intensity, which shows that difference is mainly occurring from macroelement. From the results, the OpenSees implementation shows intensity dependency and frequency dependency of the integrated models are working correctly.

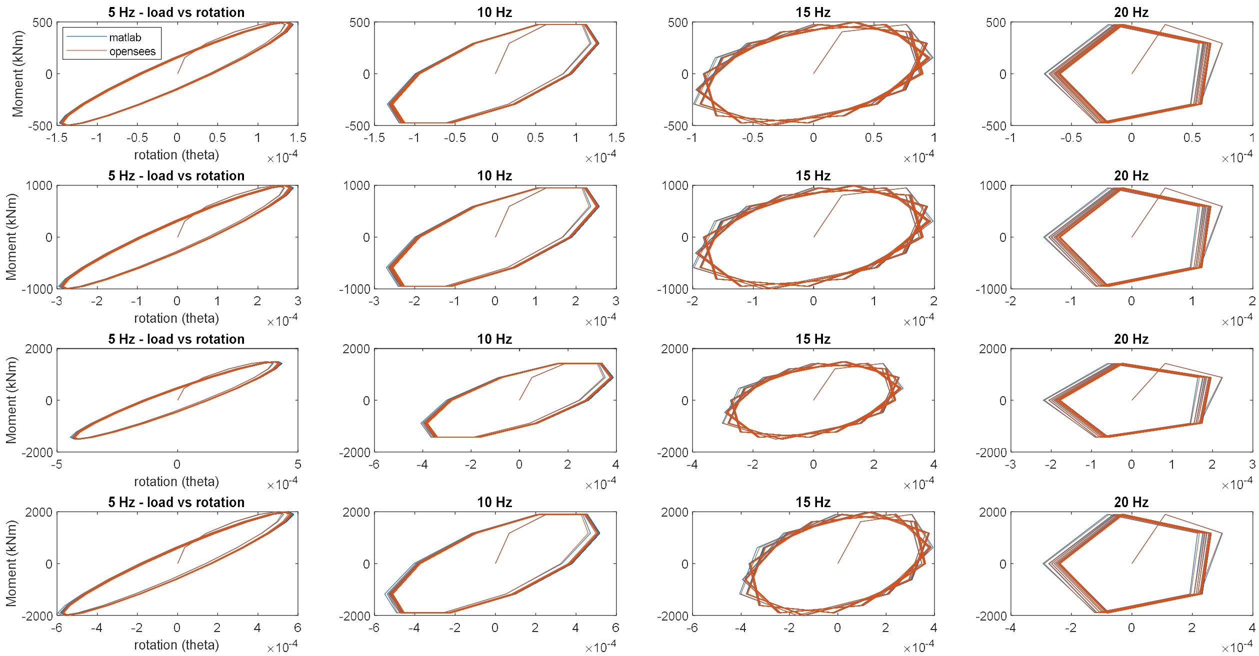


Figure 5. Verification result for nonlinear behavior with MATLAB (blue) and OpenSees (red) implementation

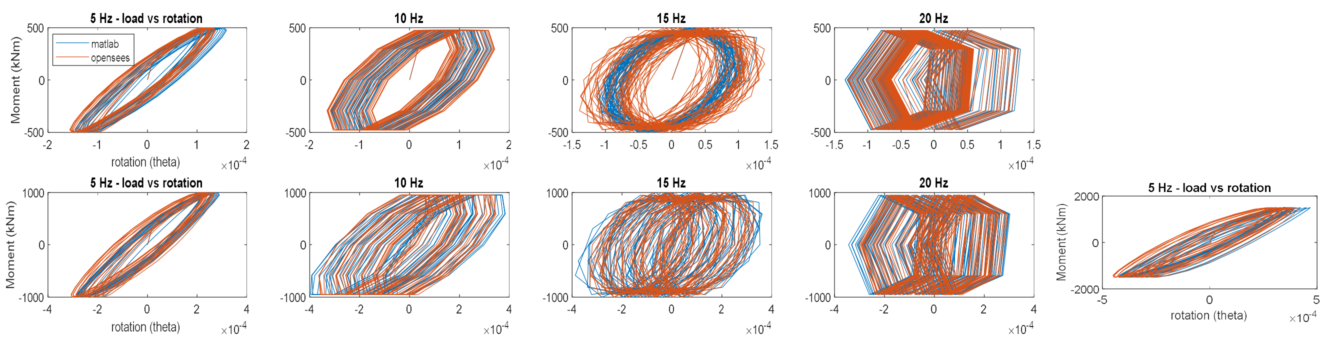


Figure 6. Verification result for nonlinear and uplift behavior of soil-foundation system with MATLAB (blue) and OpenSees (red) implementation

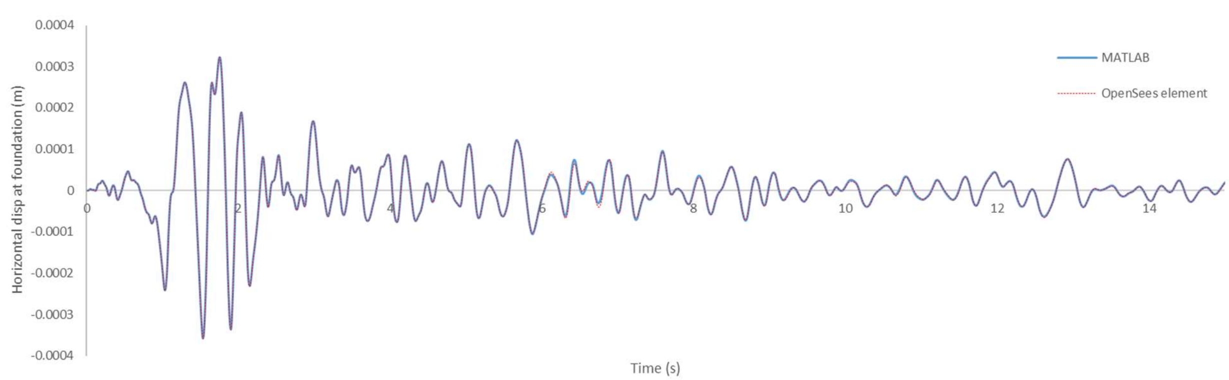


Figure 7. Nonlinear behavior of soil-foundation system subjected to Hollister excitation

Figure 8 shows horizontal displacement at the foundation when the earthquake excitation is applied at the foundation. As it is shown in the results for the analysis, the nonlinearity along with frequency dependency of the element is in good agreement with the integration method implemented in MATLAB.

CONCLUSIONS

Researchers and engineers have been investigating extensively on soil-structure interaction to reduce seismic demands to the structure with comprehensive understanding of the SSI behavior. One of the typical methods in analyzing seismic behavior of soil foundation system is to use FEM which is computationally expensive and involves significant modelling efforts. On the other hand, simplified models such as lumped spring model and nonlinear constitutive models provide efficient analysis but most of the simplified models do not have the capabilities of capturing frequency-dependency of soil simultaneously. With the use of the integration model, nonlinear constitutive model and frequency-dependent transformation model can be incorporated simultaneously in time domain analysis. From the integration model, the motivation in the paper is to provide users with accessibility of this element to the open source software, OpenSees. The implementation of this element is verified against integrated model with varying intensities and frequencies of the bridge pier model and the results are in good agreement. Future scope of work regarding the implementation is to provide users with suggested values for parameters in macroelement and provide more available options in dynamic impedance of soil domain.

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